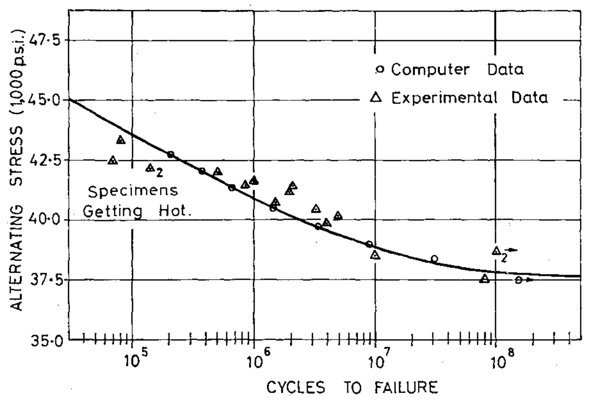

Fig. 1 Comparison of computer results with experimental data.

based on the classical strength theories are not adequate to define this physical condition. Feltner and Morrow[10] have assumed that a logarithmic plot of static true stress vs true plastic strain is valid when extrapolated back into the elastic stress region and have calculated the plastic energy dissipated per cycle using this empirical relationship. Such extrapolation techniques are not general enough to cover the microscopic elastic-plastic cases as the problem is of a statistical nature. The occurrence of plasticity in the form of isolated pockets and the extent of inelastic deformation of plastic elements are probabilistic events dependent upon the microstructure of the material. This requires a statistical approach to the strength of materials and certain microstructure sensitive experimental techniques[6,7] to define the value of the critical stress (the true elastic limit) and, hence, the statistical functions.

The dispersion of the values of stresses, strains, and elastic limits of the microelements were considered by a statistical analysis[5] and by employing the experimental technique in Ref. 6, a mathematical model for generating microplastic hysteresis loops was constructed.[8] In this analysis, the plastic strain energy dissipated per cycle was determined as a "probabilistic" quantity which cannot be arrived at by formulations based on the bulk properties of material alone. The fatigue failure was assumed to occur when this energy accumulated to a threshold value—equal to the area under the true stress—true strain diagram.[10,11]

The analysis of the plastic strain energy absorbed per cycle and the summing of this energy to fracture was carried out by computer and the points obtained compared extremely well with the experimental points. This method of analysis was successfully applied to six types of steels,[1,9] two of which have already been reported in Ref. 1 (and one is shown in Fig. 1).

Dr. Knackstedt's suggestion that the fatigue life is not connected with the total amount of energy expended during fatigue testing is very true and as our results indicate, a plastic energy approach is a very successful form of analysis provided that the mathematical expressions are constructed so as to represent the actual physics of the phenomenon at a microstructural level. Furthermore, the correlation between the true energy to static fracture and the total energy to fatigue fracture indicates a connection between the two which is inexplicable at present. Though this assumption does not explain the mechanisms of fatigue, nevertheless, both forms of fracture represent a certain amount of work that has to be done to sever the atomic bonds of the metal where the means are different but the end results are the same. We are sure that it will be a valuable contribution if Dr. Knackstedt can establish the validity of this assumption.

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Reply by Author to Comments by A. Esin and W. J. D. Jones

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THE Comment by A. Esin and W. J. D. Jones[1] is well taken, as far as they point out that and how the mechanisms of damping are much more involved than can be deduced from an analysis of tensile behavior alone. However, I did not seek to verify a failure mechanism, but rather a bulk material property.

As may be derived from the Introduction of the subject paper,[2] the mathematical model of the cyclic stress-strain relationship was established for the purpose of arriving at an analytical expression which could be used to derive numerical values of the total mechanical work that would be transferred to the specimen. I think there can be no doubt that the area enveloped by the measured hysteresis loop, representing the cyclic engineering stress-engineering strain relationship, determines this work per unit of the initial volume and cycle. By comparing the hysteresis loop with the equivalent ellipse, the order of magnitude of the exponents m or n of the plastic term in strain- or stress-controlled cyclic tests could be derived.

The measured heat produced in the specimen from the mechanical work transferred to the specimen must be subtracted in order to find the work that in Pöschl's assumption is responsible for the fatigue failure. But, still at high stresses the area of the hysteresis loop cannot then be used as an index of fatigue damage.

Note: On p. 1824 of the original article, the last equation in Sec. 222 should read:

$$\sigma_0 = \tfrac{1}{3}[2\sigma_0' + (d\sigma_0'/d\epsilon_0)\,\epsilon_0]$$

Received January 12, 1967.

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Addendum: "Performance-Weight Relations and Shape Parameters for Maxwell Structures"

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[AIAA J. **5**, 367–369 (1967)]

After the submission of this Technical Note, the author has become aware of substantial significant work on "Maxwell-Mitchell" structures conducted primarily in Great Britain. The following bibliography is presented for those readers interested to acquaint themselves further with the subject.

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Received February 24, 1967.
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Erratum: "Flame Spreading over the Surface of Igniting Solid Rocket Propellants and Propellant Ingredients"

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[AIAA J. **5**, 265–271 (1967)]

The captions under the photographs in Fig. 13 are reversed.

Received March 1, 1967.